PROCEEDINGS OF THE MATHEMATICS IN INDUSTRY STUDY GROUP

2017

Mathematics in Industry Study Group South Africa
MISGSA 2017

The writing of a Technical Report for the Proceedings of the MISGSA was coordinated by the moderator of the problem. Sections of the Report were written by the moderator and by other members of the study group who worked on the problem.

The Editor of the Proceedings was

Prof D P Mason (University of the Witwatersrand, Johannesburg)

The Technical Reports were submitted to the Editor. Each Report was refereed by one referee. On the recommendation of the referees the Reports were accepted for the Proceedings subject to corrections and minor revisions. The Editor would like to thank the referees for their assistance by refereeing the Reports for the Proceedings.

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PREFACE

The fourteenth Mathematics in Industry Study Group (MISG) in South Africa was held in the African Institute for Mathematical Sciences, Muizenberg, Cape Town, from Monday 16 January to Friday 20 January 2017.

The total number of registered participants at the MISG was seventy-six. There were eighteen Academic Staff, two Postdoctoral Fellows, fifty Graduate Students and six Industry Representatives. The invited guests were:

Graeme Hocking	Murdoch University, Western Australia, Australia
Neville Fowkes	University of Western Australia, Australia
Sarah Mitchell	University of Limerick, Ireland
Tim Myers	Centre de Recerca Matematica, Barcelona, Spain
Colin Pleasse	Oxford University, United Kingdom
Alfred Owoloko	Covenant University, Nigeria

The South African Universities and Institutes which were represented were:

African Institute for Mathematical Sciences
Council for Scientific and Industrial Research (CSIR)
North-West University
Rhodes University
University of Cape Town
University of Johannesburg
University of KwaZulu-Natal
University of Pretoria
University of South Africa (UNISA)
University of Stellenbosch
University of Western Cape
University of the Witwatersrand

The MISG meeting was opened by Professor Barry Green, Director of the African Institute for Mathematical Sciences.

The MISG followed the established format for Study Group meetings held throughout the world. South African industry had been approached to submit problems during 2016. Seven problems were submitted. On Monday morning each Industry Representative made a twenty-five minute presentation in which the problem was described and outlined. The academics and graduate students then split into small study groups and worked on the problems of their choice. Some participants worked on one problem while others moved between problems and made contributions to several problems. Each problem was co-ordinated by a senior moderator and one or more student moderators. The role of the senior moderator was to co-ordinate the research on the problem during the week of the meeting and also to do preparatory work including literature searches before the meeting. The main function of the student moderators was to present short reports at the end of each working day on the progress made that day. The moderators were in contact with the Industry Representatives throughout the meeting. On Friday morning there was a full report back session to industry. Each senior moderator, with assistance from the student moderators, made a twenty-five minute presentation, summing up the progress made and the results that were obtained. Each Industry Representative then had five minutes to comment on the progress and the results which were reported. The MISG ended at lunch time on Friday.

The MISG was preceded by a Graduate Workshop from Wednesday 11 January to Saturday 14 January 2017. The objective of the graduate Workshop is to provide the graduate students with the necessary background to make a positive contribution to the MISG the following week. The students were given hands-on experience at working in small groups on problems of industrial origin, some of which were presented at previous MISG meetings, at interacting scientifically and at presenting oral reports on their findings. Five problems were presented to the graduate students. The problems and the presenters were:

On-line sales and data analytics	Jeff Sanders, African Institute for Mathematical Sciences and University of Stellenbosch
Rhino relocation	Neville Fowkes

strategies for survival	University of Western Australia
The laminar wake behind a self-propelled body	Ashleigh Hutchinson University of the Witwatersrand
Flaws in plate glass	Neville Fowkes University of Western Australia
Dimensional analysis	David Mason University to the Witwatersrand

The graduate students worked in small study groups on the problem of their choice. Each group presented their results at a report back session on Saturday afternoon.

The sponsors of the Graduate Workshop and the MISG were:

- Hermann Ohlthaver Trust
- African Institute for Mathematical Sciences
- Centre of Excellence in Mathematical and Statistical Sciences

We thank the sponsors without whose support the Graduate Workshop and the MISG could not have taken place.

STUDY GROUP

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PROBLEMS

MODELLING OF FRACTURE DEVELOPMENT (INITIATION AND PROPAGATION) IN ROCK BASED ON THE EXTENSION STRAIN FRAILURE CRITERION

Industry: Mining

Industry Representative:

Richard Stacey, School of Mining Engineering, University of the Witwatersrand, Johannesburg.

Problem Statement

The complex behaviours of rocks and rock masses have, for long, presented paradoxes to the rock engineer, one of which is the fracturing of seemingly strong rock under low stress conditions. These conditions often occur near excavation boundaries where the confining stress is low. Understanding of the phenomenon of rock fracture and failure under such conditions is important for prediction of failure around excavations and for design of rock support to inhibit or prevent instability. A simple extension strain criterion for fracture of rock arose from such conditions: in the analysis of fracturing around a deep level tunnel boring situation, a limiting tensile strain criterion was found to be successful for predicting both the extents and the orientations of fractures observed around the bored tunnel. In contrast, conventional criteria for prediction of fracture and failure were unsuccessful.

It is quite logical to expect that any material, even perfect ``flawless'' material, will fail when it is stretched (i.e. subject to extension) beyond a certain limit. The extension strain criterion is correspondingly reasonable and is elegantly simple, being able to take into account the three dimensional stress condition (automatically including the intermediate principal stress σ_2) using a single equation from elastic theory.

The extension strain criterion has been used effectively in many applications, but has also received criticism. However, several recent publications, dealing with polyaxial testing of rock, have led to new insights regarding the effect of the intermediate principal stress on the failure of rock, especially for stress states in which σ_1 is reasonably close in magnitude to σ_2, facilitating growth of fracturing in the plane of the two larger principal stresses.

There are many reports of cases in which failure of excavations occurred at inexplicably low stresses, and such failure cannot be explained satisfactorily be strength-based criteria.

The extension strain criterion has proved to work well in practice for prediction of fracturing. Extension strain fractures will very commonly develop in a three-dimensionally compressive stress environment. Thus a fracture may develop but in this confined environment there is no change – the stresses remain the same, there is no change in deformation behaviour, there is no apparent failure of the rock. It is only when the confining stresses change, such as when an excavation is created, and ``unloading'' takes place, that the fracturing will be noticed – the ``unloading deformation modulus normal to the extension fractures will be less than the modulus in the loaded compressive environment. What is a satisfactory ``flow rule'' that can be used in modelling the creation of new extension fractures and propagation of the fractures, the propagation of the fractures and the subsequent development of *failure*? Would a suitable first step be the introduction of anisotropic linear elastic properties in the fractured environment?

Once such a system is shown to work, would it be successful in modelling the discrete fracture formation that often occurs in rock; the discrete slabbing that is observed in the sidewalls of tunnels in gold mines; the dicing observed in rock cores drilled in high stress environments; the exfoliation slabs observed on rock outcrops. And, what dictates the thickness of the slabs or discs?

HARD-TO-BOIL MASSECUITES

Industry: Sugar

Industry Representative:

Richard Loubser, Sugar Milling Research Institute, University of Kwa-Zulu-Natal

Problem Statement

Evaporators are used to remove most of the water from the juice extracted from sugar cane. The evaporators produce a syrup of about 60% dry solids (40% water). The syrup is then transferred to a crystallisation pan which evaporates more water in a process called *pan boiling*. Industrial crystallisation of sugar is performed in the meta-stable zone of supersaturation, i.e. where the degree of supersaturation is sufficient for sucrose to crystallise onto existing crystal surfaces, but not to spontaneously form new crystal nuclei. When the syrup concentration reaches the meta-stable supersaturaiton zone, seed crystals are added to facilitate crystallisation of the sucrose. The sucrose from the supersaturated syrup deposits onto the seed crystal surfaces, causing the crystals to grow. This process would lower the sucrose concentration of the syrup, however, the sucrose supersaturation is maintained by simultaneous addition and boiling of syrup. The two-phase mixture of sugar crystals and mother liquor is called a massecuite. If the syrup is reasonably pure in terms of sucrose content, the boiling and crystallisation process occurs fairly easily. However, the presence of certain impurities is known to cause problems with boiling and crystallisation, resulting in hard-to-boil massecuites.

The problematic impurities in sugar juice are thought to be the result of biological deterioration of sugarcane before it reaches the factory. The harvesting of the cane involves burning the loose leaves and then hand cutting the stalks. The stalks are then loaded onto tractors and taken to zones where they are loaded onto hilos for transport to the factory. Once the cane is burnt, it starts deteriorating. The microbial action causes polysaccharides to form. These compounds affect the rheological properties of the subsequent syrup. Delays between the burning/harvesting of cane and crushing of that cane in the factory result in increased levels of polysaccharides and other microbial by-products. Rain causes delays in loading of the cane after harvesting and hence deterioration of the cane before it reaches the factory.

In South Africa the harvesting season lasts from around March until December depending on the size of the sugarcane crop. November and December is the start of the rainy season. When the rain arrives, the phenomenon of `hard-to-boil' massecuites is often experienced. This phenomenon results in severe difficulty in crystallising sugar during pan boiling. Factories report that the massecuite appears not to boil under normal boiling conditions. Specifically, vapour bubbles do not form and crystal growth is significantly slowed. In addition, increased viscosity of the massecuite results in poor circulation of the massecuite in the pan, negatively influencing heat transfer, and causing problems with mechanical stirrers, if these are present in the pan.

The questions are:

In the formation of a bubble of water vapour in a massecuite as a result of heat transfer through a steel tube wall:

- Is the tendency for bubbles not to form in massecuites dominated by:
    1. Surface tension effects
    2. Viscosity effects
    3. Boiling point elevation
    4. Fouling of the tube wall with the extra poly-saccharides?
- What parameters will need to be determined to complete the model?

THE EFFECTS OF LANE POSITION IN A SWIMMING RACE

Industry: Sport

Industry Representative:

Ashleigh Hutchinson, School of Computer Science and Applied Mathematics, University of the Witwatersrand

Problem Statement

The Study Group is asked to investigate the effects of lane position in a swimming race. In swimming finals, lanes are allocated based on the heat times of the swimmers. The swimmer with the best qualifying time is allocated to lane 4. The second best is given lane 5. The third is allocated to lane 3. The fourth is positioned in lane 6 and so on.

There are generally 10 lanes in an Olympic swimming pool. The purpose of the lane ropes is to ensure that each swimmer remains in the designated lane. This however, is not the only purpose. Behind and around a swimmer a wake is formed. When the wakes of swimmers interact, the performance of the swimmers may be affected. The lane ropes lessen this influence. However, even with the lane ropes, these wakes still interact. The combined wake may affect swimmers on the outermost lanes differently to those on the innermost lanes.

The aim of this investigation is to study the interacting self-propelled wakes of the swimmers and to determine whether performance is influenced by lane position.

HYBRID INTELLIGENT CONTROLLER DESIGN FOR TWO-LINK FLEXIBLE MANIPULATORS USING MULTI-OBJECTIVE OPTIMIZATION TECHNIQUES

Industry: Manufacturing

Industry Representative:

Jimoh Pedro, School of Mechanical Engineering, University of the Witwatersrand

Problem Statement

Multi-link flexible manipulator systems (MFSs) are more agile and energy efficient than their rigid counterparts. However, control of FMSs is complicated by inherent non-linearity, under-actuation and non-colocation of the actuators and sensors. Conventional control schemes are poorly suited to such complexities. However, intelligence-based control schemes have been shown capable of acceptable control of FMSs. The costs of developing these novel manipulators and intelligence-based control schemes are mitigated by the need to realise a more economically and environmentally-responsible industrial sector. In the literature, mathematical modelling and simulation of multilink FMSs is the typical approach to assessing the performance of control schemes.

The nonlinear nature of the FMS problem makes the application of computational intelligence techniques like neural network, fuzzy logic and evolutionary algorithms readily attractive for its control, especially when popular modern control methods like H_2 and H_∞ often result in high-order problems that are difficult to solve. The establishment of system stability is relatively more challenging when applying computational intelligence-based control techniques.

The Study Group is asked to investigate the aptitude of three control schemes: conventional proportional+integral+derivative (PID) and intelligent PID-iterative learning control (ILC) and PID-neural network (NN) in controlling two-link flexible robotic manipulators. The control task of tracking sinusoidal waves and vibration suppression should be investigated.

The challenges that could be encountered include the following:

1. The system is a multi-degree-of-freedom problem with inter-coupling of the dynamic modes (rigid and flexible modes).
2. Conflicting design objectives requiring trade-offs,
3. Multi-objective and multivariable optimization.
4. Robustness to system nonlinearities, parameter variations and uncertainties.

The Study Group is asked to undertake the following task.

1. Design suitable conventional PID and intelligent PID-ILC and PID-NN controllers.
2. Apply an appropriate algorithm such as: particle swarm optimization (PSO), genetic algorithm (GA) and differential evolution (DE), to select the optimal gains for designed controllers.

STOCKPILING OF BAGASSE: THE HAZARD OF SPONTANOUS COMBUSTION

Industry: Sugar

Industry Representative:

Richard Loubser, Sugar Milling Research Institute, University of Kwa-Zulu-Natal

Problem Statement

Bagasse is the fibrous residue from sugar cane after the sugar containing juice has been extracted. The bagasse contains about 50% moisture after it is squeezed in a drying mill. Burning the bagasse in a boiler has the potential to produce more thermal energy than is required for the processing of the juice to make sugar. This excess energy can be used to produce electricity. The length of the milling season can be as short as four months in some countries, extending to eleven months in others. Investment in saleable power generation equipment is only feasible if power can be produced throughout the year, consequently, it is essential to store bagasse for the off-crop period when no cane is processed by the factory.

During storage, the bagasse undergoes an exothermic oxidation process. The heat generated has the potential to cause the bagasse to spontaneously ignite. This is a complex series of processes that depend on factors such as moisture content, ambient temperature, thermal conductivity and gas diffusion rates. Some sugar mills have resorted to increasing the moisture content to 60% to reduce the risk of spontaneous ignition. This practice, however, has a negative impact on the net calorific value of the bagasse.

At the 2016 MISG, a full mathematical model of spontaneous combustion of bagasse during storage was considered. Unfortunately, there was insufficient time to interpret the results into a form that could be used as a generalised guideline for the storage of bagasse.

The anticipated outcome from the study this year is an extension of the previous work in the form of a guideline that can be used in the design and operation of a bagasse storage facility. The guideline needs to advise how to minimise the calorific value losses while mitigating the hazards of spontaneous combustion.

MEASURING THE TRUE ECONOMIC COST OF HAZARDOUS LANDFILL SITES

Industry: Industrial waste disposal

Industry Representative:

Viroshan Naicker, Department of Mathematics, Rhodes University

Problem Statement

Poorly managed and regulated hazardous landfill sites represent a serious challenge to public health. As toxins accumulate at the site and as the site ages air and ground water pollution levels increase adversely affecting the environment around the site and imposing a negative economic externality on the neighbouring communities.

In South Africa, where legislation and monitoring tends to be relatively weak, landfill owners (usually large multinationals) are comfortable paying minimal fines as long as the landfill sites remain open and business can continue as usual.

The effect is that these owners are passing the true economic costs of their business to other parties and society as a whole. These costs include: long term environmental degradation, chronic health management costs, clean-up costs, potential loss of income for businesses near the landfill site as well as the decline of nearby property values.

Furthermore, these effects are cumulative: pollution levels rise, increasing and accelerating the costs to third parties. Health issues become chronic and irreversible and the costs are passed on to medical insurers and government. Property values spiral downward. Land can no longer be used for farming. Businesses and people begin to migrate from the area adding to the total economic costs.

This raises the following mathematical modelling questions for the Study Group to consider:

1. Would it be possible to model the spread and accumulation of pollution through the environment? For example, landfills are responsible both for air pollutants and for ground water pollutants. How would pollutants from a landfill site distribute themselves into the area surrounding the landfill? Airborne pollutants are necessarily carried by wind and moisture, while waterways distribute and dilute toxins from pollution sites.
2. Would it be possible to measure the economic costs of localized pollution? These include, but are not limited too, the above mentioned public health and medical expenditures, decline in property prices, loss of income and viable agricultural land and environmental clean-up costs?

OPTIMAL MOVIE SCHEDULING

Industry: Entertainment

Industry Representative:

Obakeng Moepya, Isazi Consulting, Johannesburg

Problem Statement

The Study Group will be given a list of movies with associated scores and a list of hourly scores representing peak times. It will be asked to create a month long schedule that maximises viewership.

There are numerous constraints. Only one movie can be shown at a time. There is a minimum and maximum number of runs of a movie in a month. There is a minimum movie repeat time of 48 hours and there are acceptable playing hours due, for example, to age restrictions.

The movie data includes the movie duration and one-hour time slots with their scores. There are channel-specific rules, such as peak and off-peak gaps for advertisements.

There are several challenges. The constraints are complicated, there are variations in movie duration, there are large data sets with about 100 movies and ill defined data due to missing values and inconsistent data types.

There is a current solution obtained using simulated annealing. The objectives set for the Study Group are to improve the optimization algorithm and remodel the prediction mechanism.